

###### FURTHER MATHEMATICS

**TRIAL EXAMINATION 1**

**SOLUTIONS**

**2019**

###### SECTION A SECTION B

(answers) (answers)

# Core Module 1 Module 2 Module 3 Module 4

**Matrices Networks Geometry Graphs**

**& & &**

**decision maths trigonometry relations**

**1.** D **13.** C **1.** C **1.** C **1.** A **1.** A

**2**. D **14**. A **2.** E **2.** D **2.** D **2.** D

**3**. E **15.** B **3.** A **3.** C **3.** A **3.** D

**4.** B **16.** D **4.** B **4.** E **4.** E **4.** B

**5.** A **17.** C **5.** D **5.** E **5.** D **5.** C

**6.** C **18.** A **6.** A **6.** D **6.** E **6.** C

**7.** C **19.** E **7.** A **7.** B **7.** D **7.** B

**8.** A **20.** C **8.** D **8.** E **8.** B **8.** E

**9.** D **21.** C

**10.** B **22.** B

**11.** E **23.** D

**12.** E **24.** B

# SECTION A – Core - solutions

# Data analysis

# Question 1

The data values tail off towards the higher values and hence the distribution is positively skewed. There are no outliers.

The answer is D.

**Question 2**

There are 26 data values greater than 170.

 The closest answer is 72%.

The answer is D.

**Question 3**



 (Note that the maximum occurs at what appears to be an

outlier, not at the end of the upper whisker.)

The answer is E.

**Question 4**



and so on.

We are looking for  which equals 4.

There are  artworks where .



The answer is B.

**Question 5**

Draw a diagram.



Using the 68-95-99.7% rule,

together with the symmetry of

the normal curve,



We require 18.5% in total.



The closest answer is 65.

The answer is A.

**Question 6**



Draw a diagram.

We know from the 68-95-99.7% rule

that 95% of the values lie between

the two standard deviations either side

of the mean.

Because of the symmetry of the normal

curve, this means that 2.5% lie above

2 standard deviations above the mean. The shaded area indicates this area of 95% and 2.5%.

So the randomly selected adult must have a standardized body temperature of .

The answer is C.

**Question 7**

Option A is true. Note that the outlier for town *A* is taken as the minimum value of town *A*’s distribution.



Option B, is true since data values in the box and top whisker lie well above 120.

Option C is **not** true since the ranges and interquartile ranges of the two distributions are about the same.

The answer is C.

**Question 8**

Key the data into your CAS and go to the linear regression line equation option.

This option not only gives you the coefficients of the linear regression line equation (*a* and *b*) it also gives you the value of *r*, Pearson’s correlation coefficient.

So  The closest value is .

Note that when you are just finding the value of *r*, it doesn’t matter whether you make the variable *age* the *x* or *y* variable, as you will get the same result. It WOULD matter however if you had been asked to find the equation of the regression line.

The answer is A.

**Question 9**

Choose any two points that lie on the line, for example .



The equation of a line with this gradient, through the point  is



(If we use the other point we get the same answer i.e.



The closest answer is 

The answer is D.

**Question 10**

The value of *r* would change if the data point (13, 92) was changed to (31, 92).

The interquartile range of cellaring time originally would have been .

After the change, it would have still been , so this is the statistic that doesn’t change.

The answer is B.

**Question 11**

From the formula sheet we have



The closest answer is 0.97.

The answer is E.

**Question 12**

The pattern is **best** described as having seasonality with a downward trend.

The answer is E.

**Question 13**

The data point representing the number of tourists in September 2013 is circled in the diagram below.



The three months prior to that are in a box and the three months after are in a triangle.

The median number of tourists for these 7 data points occurs for August 2013 and is approximately 8900.

The answer is C.

**Question 14**

A seasonal index of 0.96 tells us that the total attendances that week (i.e. Week 2) are 4% less than the weekly average.

The answer is A.

**Question 15**

The sum of the long-term average attendances is 134 400.

The average weekly long-term average is .

The seasonal index for Week 4 is 

The closest answer is 0.82.

The answer is B.

**Question 16**





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The answer is D.

**Recursion and financial modelling**

**Question 17**

Generate the sequence on your CAS.

22 000, 20 764, 19 513.17, 18 247.33, 16 966.29



The answer is C.

**Question 18**

This model shows an investment that grows by a constant $130 each year. Because it is constant each year we must have simple interest and the rate must be .

So option A gives the correct answer.

For options C and D, that is for compound interest, the recurrence relation rule would be of the form  where *c* is a constant.

For option E, that is for an annuity investment, the recurrence relation rule would be of the form  where *c* and *d* are constants.

The answer is A.

**Question 19**

For the annuity investment, both the interest and the payment are added to the balance.



The answer is E.

**Question 20**



The dishwasher went through 15 000 wash cycles over the five years.



The dishwasher was depreciated by $480 over the five years.

Its purchase price was therefore .

The answer is C.

**Question 21**

 because this is the amount Jane initially invests.

Annual interest of 4.2% represents monthly interest of  and this is equal to 0.0035.

The rule is therefore .

The recurrence relation is 

The answer is C.

**Question 22**

A perpetuity is an annuity whereby the payment received is equal to the interest earned. This means that the value of the perpetuity is constant. Only option B shows this.

The answer is B.

**Question 23**

Method 1 – using a recurrence relation

Janine has a reducing balance loan and we can write a recurrence relation for it



.

Generating this sequence on your CAS gives 

So the balance will first drop below $50 000 after 19 months.

The answer is D.

Method 2 – using TVM



Since payments need to be integer (whole number) values, the next value of N is 19.

You can double-check this by putting in  into TVM and find the FV.

It will be – 49 789.755… i.e. the balance has just dropped below $50 000.

The answer is D.

**Question 24**

Use TVM to first find the interest rate.

Note that this is a compound interest investment and NOT an annuity investment, in other words , because Sophie is not making quarterly payments to her investment like you do with an annuity investment.

Note that there is a difference of 3 years or  quarters between the two balances we are given.



Use this rate to find what the PV was initially using the 2 year balance.



So the initial investment was closest to $8300.

Note that you can confirm this by using the 5 year balance as well, i.e.



The answer is B.

**SECTION B - Modules**

**Module 1 – Matrices**

**Question 1**

A triangular matrix has all zeros either above the leading diagonal or below the leading diagonal. Reject option A.

A zero matrix contains only zeros. Reject option B.

The matrix is binary i.e. it contains only 0’s and 1’s.

Note, it is not a permutation matrix because it has two 1’s in row 4 and two in column 4.

The answer is C.

**Question 2**

*M* is a  matrix.

*P* is a  matrix.

*Q* is a  matrix.

For the matrix product to be defined the number of columns of the first matrix must equal the number of rows of the second.

For option A, 



This is not defined since matrix *M* has 1 column and matrix *P* has 2 rows.

For option B, 



This is not defined since matrix *M* has 1 column and matrix *Q* has 2 rows.

For option C, 



This is not defined since matrix *P* has 1 column and matrix *M* has 3 rows.

For option D, 



This is not defined since matrix *P* has 1 column and matrix *Q* has 2 rows.

The answer is E.

**Question 3**

Expressing this system as a matrix equation, we have



There is no unique solution when the determinant of the  matrix equals zero i.e.



The answer is A.

**Question 4**

Orazio has three one-step dominances, one over Nick, one over Mary and one over Pyang.

He has only **one** two-step dominances over Nick however, ie Orazio over Pyang over Nick*.*

The answer is B.

**Question 5**

Let  be the element in the *i*th row and *j*th column of matrix *F.*

Note that  and since *S* and *P* both have transitions to themselves of 0.1 then the first column and row could be *S* or *P*.

Note also that  and only *P* has a transition of 0.1 (to *C*).

So the first column and row of matrix *F* should be labelled *P*.

The second column and row must therefore be labelled *C*.

On the diagram we see that *C* has a transition of 0.5 to *Y* and since , then row 3 and hence column 3 must be *Y.* The last column and row must therefore be *S*.

The matrix should be



The correct order is *P, C, Y, S.*

The answer is D.

**Question 6**



So working backwards,



The two simultaneous linear equations are



Only the second option is given.

The answer is A.

**Question 7**

Since *C* is a diagonal matrix, *C* must be a square matrix so reject E.

Diagonal matrices have zeros everywhere except along their leading diagonal. Reject option B.

Since , then a decrease of 5% can be obtained by multiplying by 0.95.

Similarly an increase of 3% and of 2% is obtained by multiplying a number by 1.03 and 1.02 respectively.

The answer is A.

**Question 8**



In week three, 156 visitors are predicted to visit the Charlesville skate park.

The answer is D.

**Module 2 – Networks and decision mathematics**

**Question 1**

A complete graph has an edge between each pair of vertices as shown in the diagram.

The adjacency matrix is





The answer is C.

**Question 2**

Dijkstra’s algorithm is used to find the shortest path between two vertices so option D is correct.

Note that for options A and E we would need a directed network.

The answer is D.

**Question 3**

For a graph to contain an Eularian trail it must have exactly two vertices with odd degrees. The first graph does but the second doesn’t. Reject option A.

For a graph to contain an Eularian circuit it must have all vertices with even degrees. The first graph doesn’t and the second does. Reject option B.

Both graphs have Hamiltonian paths.

The answer is C.

**Question 4**

A planar graph can be drawn so that no edges cross.

This can be done for options A - D but not for option E as shown below.



The answer is E.

**Question 5**

A forward scan is shown below.



The minimum completion time is 17 hours.

The answer is E.

**Question 6**

The critical path is *A C E F* and this gives a completion time of 18 days.

The only other path is *A B D F* and this gives a completion time of 17 days.

Activities *A* and *F* should be reduced which will reduce the completion time of both paths by 2 days.

If activities *C* and *E* are each reduced by 1 day then path *A C E F* will take 14 days.

By reducing one of *B* and *D*, the path *A B D F* will also take 14 days.

So reducing five activities will achieve the greatest reduction in time for minimum cost.

The answer is D.

**Question 7**



The three possible minimal spanning trees are shown above.

The answer is B.

# Question 8

If , then jobs 1 and 3 would be allocated to Chloe and Doc and there wouldn’t be enough hours for jobs 2 and 4 to make the total for the four jobs up to 10. Reject option A.

If , using the Hungarian algorithm,

|  |  |  |  |
| --- | --- | --- | --- |
| 3 | 2 | 4 | 3 |
| 4 | 3 | 4 | 2 |
| 3 | 2 | 2 | 3 |
| 2 | 3 | 3 | 4 |

After row reduction.

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 0 | 2 | 1 |
| 2 | 1 | 2 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 2 |

There is only one possible allocation here:

Amy 🡺 job 2

Barney 🡺 job 4

Chloe 🡺 job 3

Doc 🡺 job 1

For this allocation the total time required to complete all four jobs is .

Reject option B.

If , we have

|  |  |  |  |
| --- | --- | --- | --- |
| 3 | 2 | 4 | 3 |
| 4 | 3 | 4 | 2 |
| 3 | 2 | 3 | 3 |
| 3 | 3 | 3 | 4 |

By observation, possible allocations with a total time of 10 hours are

Amy 🡺 job 2 Amy 🡺 job 2

Barney 🡺 job 4 or Barney 🡺 job 4

Chloe 🡺 job 1 Chloe 🡺 job 3

Doc 🡺 job 3 Doc 🡺 job 1

or Amy 🡺 job 1

Barney 🡺 job 4

Chloe 🡺 job 2

Doc 🡺 job 3

There may be more possible allocations but there are certainly more than two and the manager found that there were only two possible allocations.

So *k* must be more than three.

Note that the option “at least 3” includes 3 so it cannot be that.

So we reject option C and option D.

The only option left is E.

The answer is E.

**Module 3 – Geometry and trigonometry**

**Question 1**



The closest answer is 23.4.

The answer is A.

**Question 2**

We use the first value, i.e. the latitude, to decide which location is closest to the south pole which has a latitude of 90°S.

The closest location is 76°S 5°E.

The answer is D.



**Question 3**

In the right angle triangle shown,



The closest bearing is 034°.

The answer is A.

**Question 4**



The closest answer is 285 cm3.

The answer is E.

**Question 5**



’s *ABC* and *DEC* are similar so



The answer is D.

**Question 6**

Note that of the four slant sides, two have the same area and the other two have a different area. For the front slant face, using Heron’s formula (formula sheet), we have



For the side slant face,





The answer is E.

**Question 7**







The closest answer is 4.1 metres.

The answer is D.

# Question 8

Draw a diagram.





The closest value of *x* is 42.3.

The answer is B.

# Module 4 – Graphs and relations

**Question 1**

The height is decreasing at the greatest rate when the gradient is steepest.

From the graph, this occurs between 8.30am and 9am.

The answer is A.

**Question 2**



Only the point  produces a value less than six.

The answer is D.

**Question 3**

The relationship between *y* and ** is linear i.e.  where *m* is the gradient of the straight line shown.

The gradient of the line is .

i.e. 



The answer is D.

**Question 4**

The rule is



The answer is B.

**Question 5**

Option A is true.

Option B is true since when , Sian is 12 km from the start and Julian is 10 km from the start. Option C is not true.

The gradient of Sian’s graph is . Sian walks at 6 km/hr for the entire time.

The gradient of Julian’s graph between  is also 6.

i.e. 

The answer is C.

**Question 6**











The answer is C.

**Question 7**

Using ratios, we have





We want the values of *x* to be “at least” so use the inequality ≥



Now rearrange,



The answer is B.

# Question 8

Using a ruler, angle it on the same slope as the dotted line. Now, maintaining this slope, move the ruler away from the origin. This is known as the sliding-line method.

The last points it touches on the feasible region lie along the line segment *AB*.

So the maximum value of the objective function will occur here.

The answer is E.